

# Distribution of Active Earth Pressure on Vertical Shafts



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## ABSTRACT

This study describes the experimental program that has been conducted to investigate the distribution of earth pressure on a cylindrical wall embedded in granular material and subjected to radial displacement. A model shaft has been designed and built using mechanically adjustable segments to control the magnitude and uniformity of the wall movement during the tests. A series of experiments have been performed and the progressive changes in earth pressure along the shaft have been continuously measured. Results indicated a rapid decrease in lateral earth pressure when a small wall movement is introduced. The experimental results are also compared with some of the available theoretical solutions and the applicability of these solutions is then examined.

## RÉSUMÉ

Cette étude présente un programme expérimental élaboré afin d'étudier la distribution de la poussée des terres sur un mur cylindrique enterré dans un matériau granulaire et sujet à un déplacement radial. Dans cet esprit, une colonne constituée de segments ajustables reliés par un système mécanique a été conçue pour contrôler l'ampleur et l'uniformité du mouvement du mur durant les essais. Des séries d'expériences ont été menées et la variation progressive de la poussée des terres le long de la colonne a été mesurée de manière continue. Les résultats ont montré une réduction rapide de la poussée des terres lorsqu'un petit mouvement du mur est introduit. Les résultats expérimentaux sont également comparés à quelques solutions théoriques disponibles et l'applicabilité de celles-ci est examinée.

## 1 INTRODUCTION

Classical earth pressure theories developed by Coulomb (1776) and Rankine (1856) have been often used to estimate earth pressure on shaft constructions; however these theories were originally developed for infinitely long walls under plane strain conditions. Several attempts have been made to extend these classical earth pressure theories to study the active earth pressure against cylindrical shafts in cohesionless media. Westergaard (1940), Terzaghi (1943), and Prater (1977) used the limit equilibrium method; whereas Berezantzev (1958), Cheng & Hu (2005), Cheng et al. (2007), Liu et al. (2009) used the slip line method of analysis. To facilitate the solution of the governing equations, the above methods generally adopt a simplifying assumption related to the coefficient of lateral earth pressure on radial planes,  $\lambda$ , which is defined by the ratio between the tangential ( $\sigma_\theta$ ) and vertical ( $\sigma_v$ ) stresses (Prater, 1977). In contrast to the classical earth pressure theories, where the active earth pressure calculated using Coulomb or Rankine method are basically the same, the distributions obtained for axisymmetric conditions may differ considerably depending on the chosen method of analysis as shown in Figure 1. In addition, the required wall movement to reach the calculated pressures is yet to be understood.

Physical models have also been used by several researchers (e.g. Walz, 1973; Lade et al., 1981; König et al., 1991; Fujii et al., 1994; Herten & Pulsfort, 1999; Chun & Shin, 2006, Imamura et al., 1999) to study the changes in earth pressure due to the installation of model shafts in granular material. One of the key challenges in developing

a model shaft is the ability to simulate the radial movement of the supported soil during construction. Researchers have developed various techniques to capture these features either during or after the installation of the instrumented lining.

Walz (1973) designed a segmented lining apparatus equipped with a cutting edge piece with a recess to induce soil displacement during the shaft sinking process. Cutting edges with different recesses (0 to 5 mm) were used in the experiments. It was concluded that contact pressure acting on a shaft will significantly decrease if a small soil displacement is allowed. Lade et al. (1981) conducted a series of centrifuge tests to investigate the active earth pressure on shaft lining. A flexible tube filled with fluid was used to model the shaft structure. The excavation process was simulated by incrementally reducing the fluid level inside the lining.

Another approach to introduce the radial movement of a shaft lining involves the design of a mechanically adjustable system. Fujii et al. (1994) and Imamura et al. (1999) used an aluminum tube that has been split longitudinally in two parts. During the test, one of the parts was moved using an electric motor. Stress transducers were installed in the moving part of the lining to record the earth pressures.

Herten & Pulsfort (1999) simulated one quarter of a cylindrical shaft placed at the corner of a rectangular box that contained sandy soil. Chun & Shin (2006) conducted an experimental study of the effects of wall displacement and shaft radius on the earth pressure distribution using a mechanically adjustable semi-circular shaft. It was concluded that the soil failure surface extended a distance of approximately one radius from the outer perimeter of

the shaft. These experimental studies provided the basis for the development of a new mechanically controlled apparatus that satisfy the axisymmetric configuration and allow for the continuous measurement of the earth pressure acting on the shaft lining.

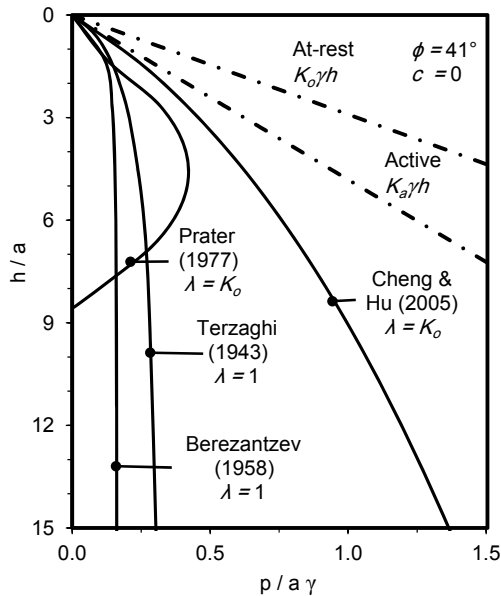


Figure 1. Earth pressure using different theoretical methods (a, shaft radius; h, height)

Table 1 summarizes the required wall displacement for establishing active conditions reported in some of the above work. Based on the experimental results, it is evident that no agreement has been reached among researchers as to the required wall movement to reach active conditions. The displacements range from 0.05% to 1.8% of the shaft height and from 0.15% to 6.6% of the shaft radius as shown in Table 1. This can be attributed to the difference in testing conditions, shaft geometry, and wall movement technique used in each study.

Table 1. Comparison of the required wall displacement for active condition

	Wall displacement, S		Sand state
	$S \geq 1\% H$	$S \geq 6.6\% a$	
Fujii et al. (1994)	$S \geq 1\% H$	$S \geq 6.6\% a$	Dense
Imamura et al. (1999)	$S = 0.2\% H$	$S = 1.6\% a$	Dense
Herten & Pulsfort (1999)	$S = 0.05\% H$	$S = 0.25\% a$	Dense
Chun & Shin (2006)	$0.6\% H < S < 1.8\% H$	$0.15\% a < S < 0.4\% a$	Dense

In this paper the experimental setup designed to investigate the active earth pressure on a cylindrical shaft is described. A segmented model shaft (1 m in length and 0.15 m in diameter) has been build to satisfy the full

axisymmetric geometry and facilitate the control of the wall displacement. The radius of the lining has been reduced in four increments, namely, 1, 2, 3 and 4 mm and the corresponding earth pressure results are summarized. In addition, the performance of some of the available analytical solutions is evaluated.

## 2 EXPERIMENTAL SETUP

The experimental setup consisted mainly of a cylindrical concrete tank (1.22 m in diameter and 1.07 m in depth) that contained the instrumented shaft. Details of the model shaft and experimental procedure are given below.

### 2.1 The model shaft

The model shaft consisted mainly of six curved steel segments as shown in Figures. 2 and 3. The segments were machined to fit into segment holders that were connected using steel hinges to hexagonal nuts. These nuts passed through a central threaded rod, extended through the shaft axis, that consisted of a right hand and left hand threaded rods joined by a collar. The basic mechanism that contracts the diameter of the apparatus is quite simple; as the axial rod was rotated the nuts moved vertically, pulling the segment holders radially inwards, and, therefore, uniformly translating the shaft lining (see Figure 2). This process leads to an inward movement of the lining segments and consequently the shaft diameter uniformly decreases.

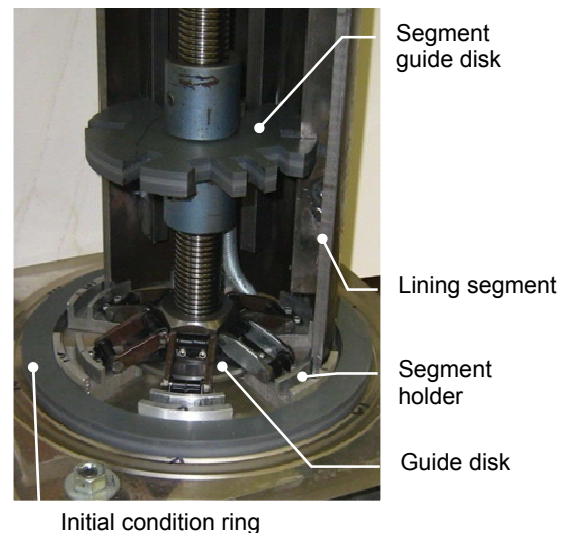


Figure 2. Details of the mechanical adjustable system

To measure the earth pressure, three load cells were installed behind one of the lining segments, with sensitive circular areas of one-inch in diameter in contact with the soil. The centers of these sensitive areas were located at distances of 84 mm, 240 mm, 490 mm below the sand

surface. Additionally, two LVDT's were used to monitor the wall movement at two locations near the top and bottom of the lining as shown in Figure. 3.

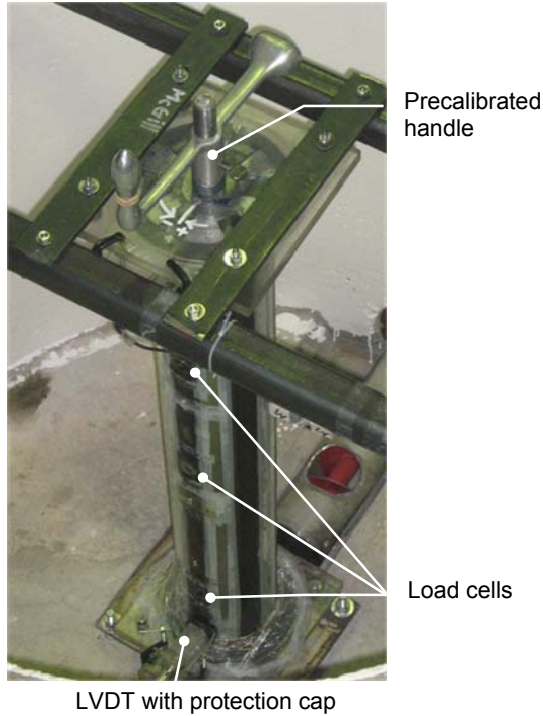


Figure 3. The installed model shaft

## 2.2 Procedure

Quartz Industrial 2075 coarse sand with no fines ( $D_{10} = 0.75$  mm;  $D_{60} = 1.75$  mm) was placed through pluviol deposition. Density cups were placed at different layers inside the tank during the sand placement. The average unit weight across the tank was found to be  $14.7 \text{ kN/m}^3$ . A summary of the sand properties is given in Table 1. After the initial readings were recorded; the shaft diameter was slowly reduced by rotating the precalibrated handle installed at the top of the shaft and the test was terminated when a maximum displacement of 4 mm was reached.

A total of 12 tests were conducted and the readings were collected for each wall movement increment (1, 2, 3, and 4 mm). The experiments were repeated three times to ensure their reproducibility and the above procedure was followed for each test. A summary of the test results is reported in the following section.

Table 2. The material properties

Specific gravity	2.65
Coefficient of uniformity ( $C_u$ )	3.6
Coefficient of curvature ( $C_c$ )	0.82
Minimum dry unit weight ( $\gamma_{min}$ )	$14.2 \text{ kN/m}^3$
Maximum dry unit weight ( $\gamma_{max}$ )	$16.4 \text{ kN/m}^3$
Experimental unit weight ( $\gamma_d$ )	$14.7 \text{ kN/m}^3$
Unified soil classification	SP
Internal friction angle ( $\phi$ )	$41^\circ$
Cohesion ( $c$ )	0 kPa

## 3 EXPERIMENTAL RESULTS AND DISCUSSION

The measured earth pressure recorded by the load cell located at  $0.84H$  below the sand surface is shown in Figure 4. The measured pressure,  $p$ , is normalized with respect to the initial pressure,  $p_0$ , and plotted on the vertical axis whereas the axisymmetric wall displacement,  $S$  (mm), is plotted on the horizontal axis. The results indicated a consistent decrease in earth pressure at the three investigated locations ( $0.24H$ ,  $0.49H$  and  $0.84H$ ) along the shaft from 100% of the initial pressure for  $S = 0$  mm to about 20% for  $S$  greater than 3 mm. It can be seen that the pressure initially decreased rapidly for small wall movements (less than 1 mm); whereas for  $S$  greater than 1 mm, the rate of reduction was less significant, and the pressure reached a constant value of approximately 20% of the initial pressure when the wall displacement was about 2 mm (about 2% of the wall height or 2.5% of the shaft radius).

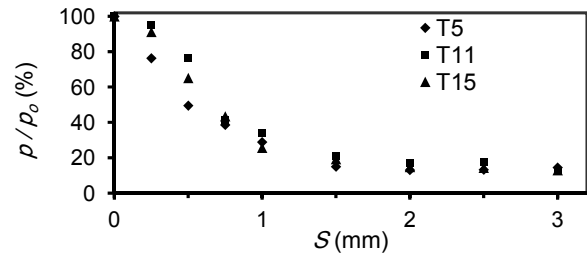


Figure 4. Normalized pressure at  $H = 0.84H$  for wall displacements up to 3 mm.

The distribution of the measured earth pressure with depth for different wall movements ( $S = 1, 2,$  and  $3$  mm) is shown in Figure 5. For  $S = 1$  mm ( $0.1\% H$ ), the lateral pressure increased from the surface up to the mid height of the shaft ( $h/a \approx 7$ ) and slightly decreased below this depth. The lateral pressure decreased with the increase of wall movement to 2 mm and the distribution became more uniform with depth. Additional increase in wall movement to 3 mm did not cause significant changes in earth pressure.

To evaluate the performance of some of the available theoretical methods, the experimental results are compared with four different solutions, namely Terzaghi (1943), Berezantzev (1958), Prater (1977) and Cheng & Hu (2005) as summarized in Figure 5.

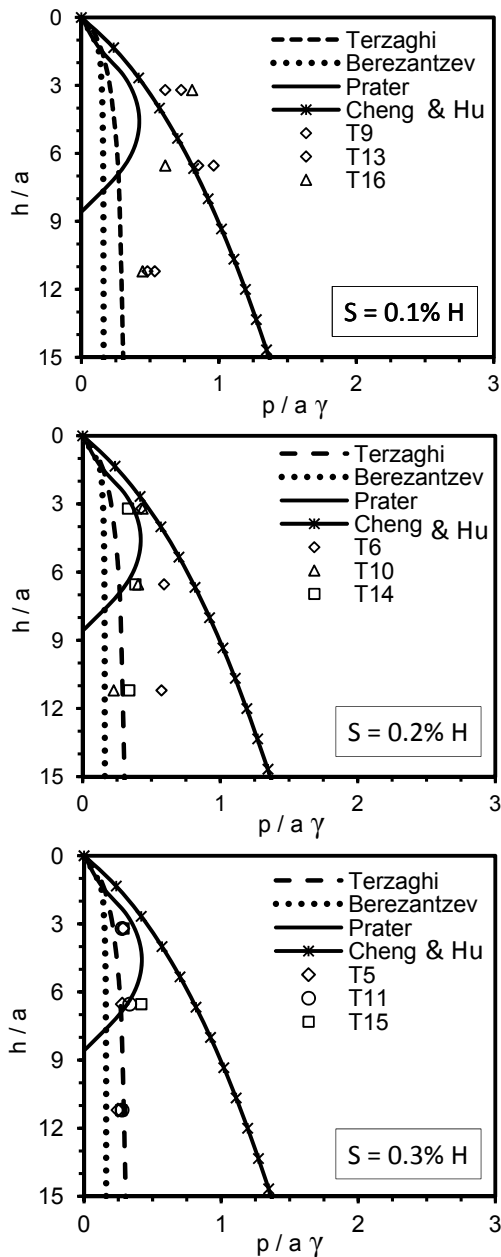


Figure 5. Comparison of the measured and theoretical earth pressure distribution along the shaft

The solutions proposed by Terzaghi (1943) and Berezantzev provide reasonable estimates for the earth pressure acting on the shaft lining once enough wall movement is allowed. This is consistent with the results of limit equilibrium and slip line methods on which these theories are based. On the other hand, the solution proposed by Prater (1977) is in agreement with the measured pressures at the upper half of the shaft, once the retained soil fully yielded. However, the solution

estimates a zero pressure at depth which is inconsistent with the experimental data.

Cheng & Hu (2005) proposed bounds for the earth pressure distribution based on slip line analysis using different values of the coefficient  $\lambda$ . The upper bound was derived using  $\lambda = K_0$ ; whereas the lower bound was based on  $\lambda = 1$  which reduces the solution to the one proposed by Berezantzev (1958). The active pressure distribution computed using  $\lambda = K_0$  agrees well with the experimental results for the upper half of the shaft when small movements are induced. However, the predicted distribution is not uniform with depth as measured. It can be observed that the earth pressures measured for wall movements greater than 0.1% of the wall height, 1 mm, agree well with the range of pressures predicted using Cheng & Hu (2005).

The above comparison highlights the importance of the relationship between the allowed soil movement around the shaft and the expected earth pressure distribution as it has been shown to have significant implication on the chosen method of analysis.

#### 4 CONCLUSIONS

Experimental investigation has been conducted to measure the changes in lateral earth pressure on a cylindrical shaft in granular material subjected to different wall movements. Earth pressure was found to decrease rapidly when a small wall movement was induced and reached a stage (about 2.5% of the shaft radius) where the pressure became independent of the wall movement.

For shafts in cohesionless soils no agreement has been reached among researchers as to the magnitude of wall movement required to reach the active condition. Based on this study, the wall movement needed to establish active condition is approximately 0.2% of the wall height; however, the reduction in pressure can reach about 80% of the initial value, in contrast to a reduction of about 40% predicted using the classical earth pressure theories. These conclusions are based on limited number of 1g tests, and therefore, field verification is needed to confirm the above findings.

#### ACKNOWLEDGEMENTS

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